Graphing Art Revisited: The Evolution of a Good Idea

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Abstract: *The Graphing Art Project encourages students to explore functions as they create art. Students write functions with domain and range restrictions that generate interesting pictures. The project is easily tailored for different levels of mathematical learners. Algebra 1 students create art using linear functions; precalculus students include conic sections, exponential, logarithmic, trigonometric functions and the greatest integer functions in their work. Students may write graphing calculator programs to reproduce and animate their art.*

Keywords. *Function, art, graphing, multiple representations*

1 Introduction

"The proposition that important mathematical ideas and methods can be taught most effectively by engaging students in work on interesting problems is one of the most attractive recommendations of recent NCTM Standards documents" (Marcus, 2003, p. 55). Such an activity appeared in Disher (1995). That article with some modification has been used in my classes every year since. The activity is particularly relevant to the study of functions and relations because it incorporates linear, quadratic, absolute value and inverse functions all in one activity. In addition to writing algebraic relations and graphing them, students need a strong understanding of domain or range to complete their work. As I read about the project and started using it with my own students, I realized that the activity could be easily tailored for different levels of mathematical learners. The project can be used in an Algebra 1 course as students learn about linear functions. On the other hand, it can be used in a Precalculus course with conic sections, exponential, logarithmic, trigonometric functions and greatest integer function. Over the years, I have had students write about their project in a journal. Students in my Honors Algebra 2 course took the project a step further, programming their graphing calculators to replicate the drawings they initially charted out with pencil and paper. A number of my students have animated their drawings using basic programming features on the graphing calculator and the *TI Graph Link*.

2 Explanation of Project

To ensure the students have learned the necessary concepts to work independently, the project starts with tasks 1 and 2. In task 1, students are provided with a list of algebraic relations and domain / range restrictions.

1.	$y = -\frac{1}{3}(x+5)^2 + 13$	$D: -8 \leq x \leq -2$
2.	$y = \frac{7}{2}x + 17$	$D: -4 \leq x \leq -2$
3.	$y = 3$	$D: -4 \leq x \leq 5$
4.	$x = -\frac{2}{9}(y-0)^2 + 7$	$R: -3 \leq y \leq 3$
5.	$x = -\frac{3}{2}\lvert y+7 \rvert + 6$	$R: -11 \leq y \leq -3$
6.	$x = 0$	$R: -15 \leq y \leq -3$
7.	$x - y = 14$	$D: -1 \leq x \leq 0$
8.	$y + 3 = 0$	$D: -5 \leq x \leq 5$
9.	$3x + 2y = -21$	$D: -7 \leq x \leq -5$
10.	$10x - 3y = -70$	$R: -7 \leq x \leq -4$
11.	$y = 10$	$R: -6 \leq y \leq -4$
12.	$y - 8 = 2\lvert x+7 \rvert$	$D: -8 \leq x \leq -6$

As students plot these relations, a familiar figure is revealed - namely, the flamingo shown in Figure 1. In the follow-up task, illustrated in Figures 2 and 3, students work backwards. First they are provided with a specific picture on a grid - in this case, the space shuttle shown in Figure 2. From the picture, students derive algebraic definitions for the numbered parts as depicted in Figure 3.

3 Student Creations

After the students work successfully with teacher-generated functions and graphs, they are ready to develop their own creations. When the students begin to create a picture, I have them draw their axes on a piece of graph paper, sketch their picture and number all of the relations. As the students begin to sketch their pictures, it is important to ensure that their pictures include a variety of different relations that are accessible to students. When students begin the project, some don't realize how much time it will take to write algebraic definitions for the relations they draw. Many students produce sketches with definitions that are too complicated to derive during the allotted time. With some assistance, however, these students revise their sketches successfully, creating novel drawings that are mathematically accessible.

Ultimately, students submit sketches that include a variety of numbered segments and curves (see Fig. 4). As I assess these sketches, I am able to find errors in students'

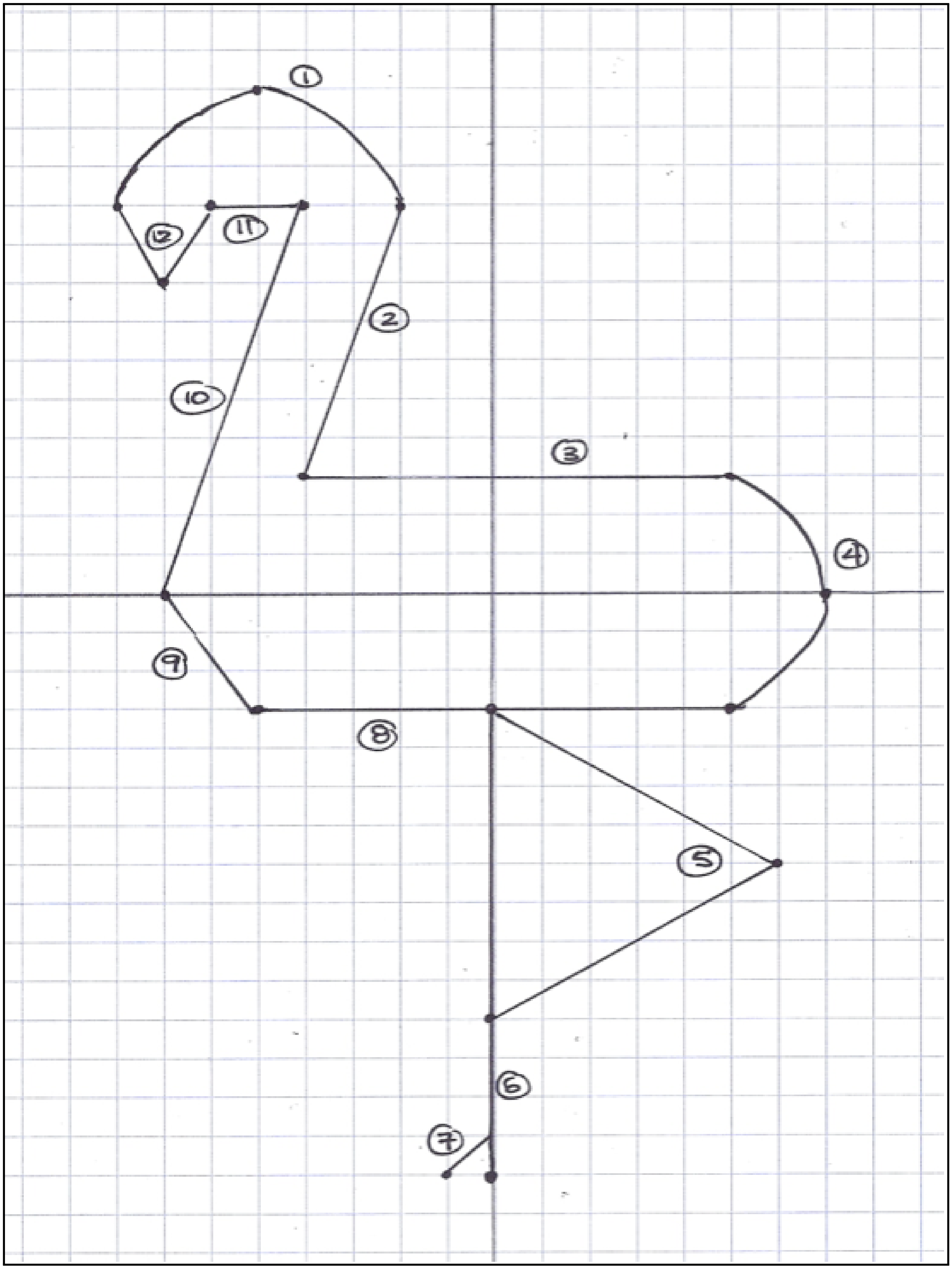


Fig. 1: *Student art generated from list of algebraic definitions*

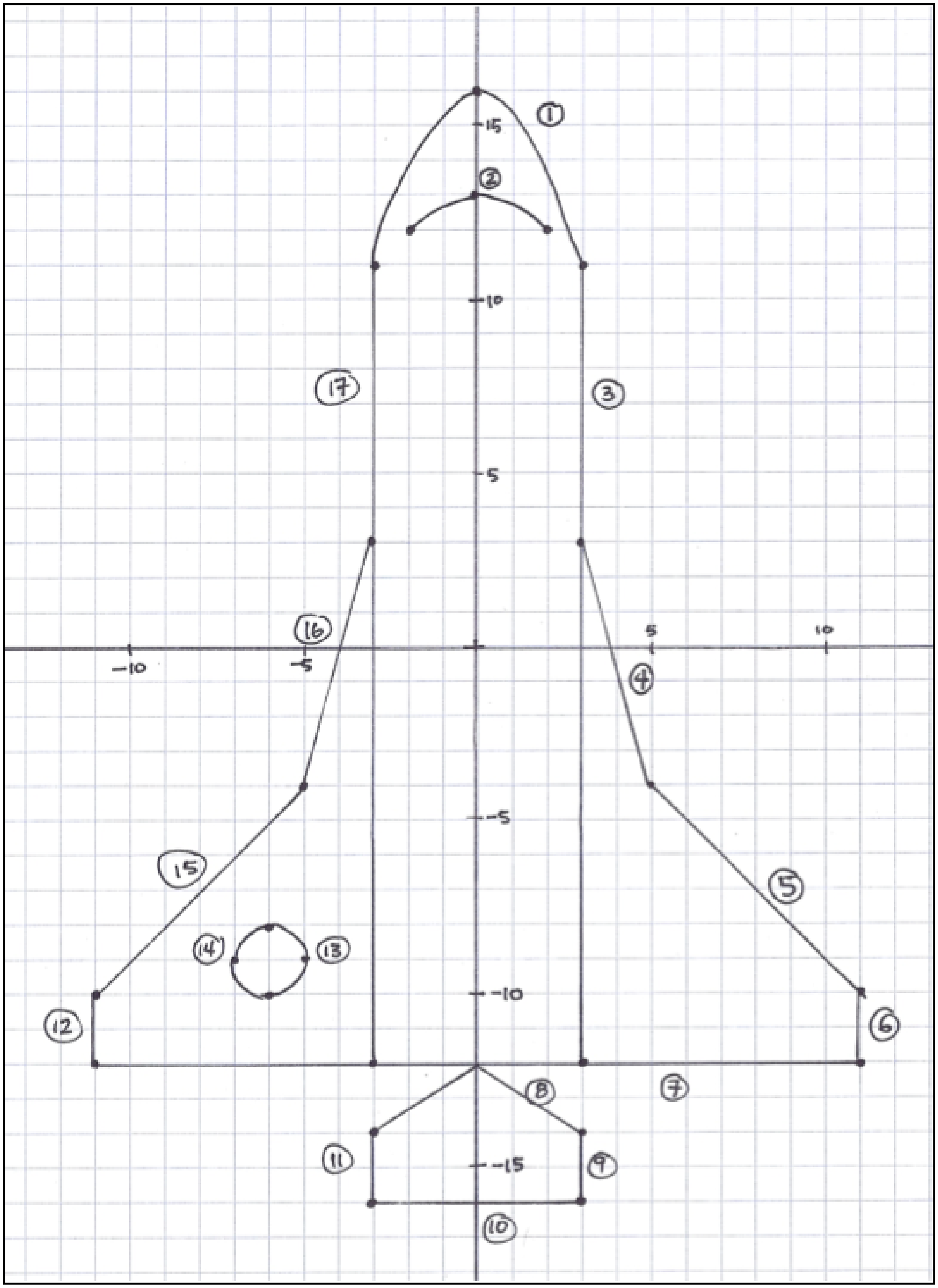
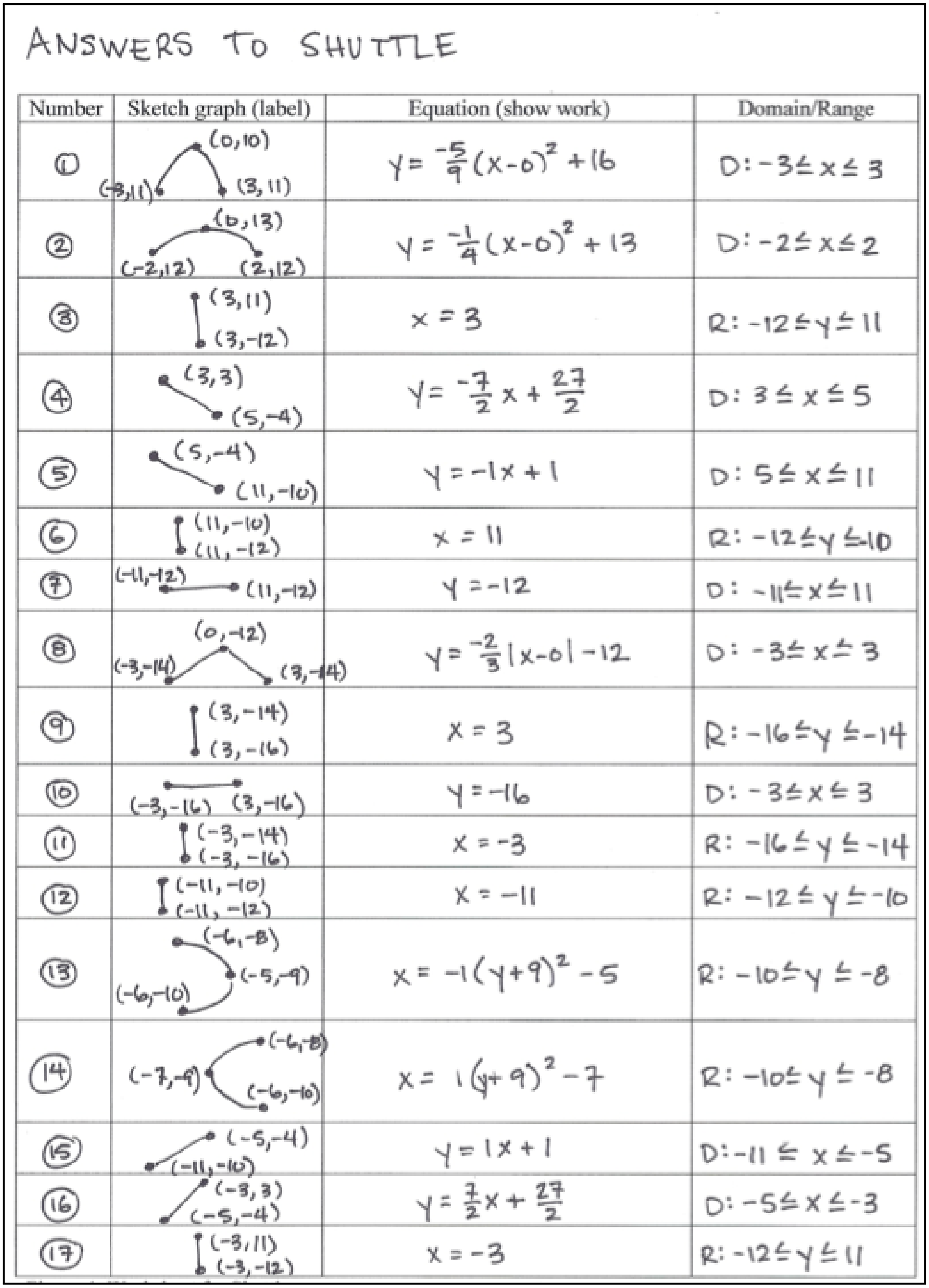


Fig. 2: *Space shuttle picture comprised of various algebraic relations*



ANSWERS TO SHUTTLE

Number	Sketch graph (label)	Equation (show work)	Domain/Range
1	(0,10) (-3,11) (3,11)	$y=\frac{-5}{9}(x-0)^2+16$	D: $-3\leq x\leq 3$
2	(0,13) (-2,12) (2,12)	$y=\frac{-1}{4}(x-0)^2+13$	D: $-2\leq x\leq 2$
3	(3,11) (3,-12)	$x=3$	R: $-12\leq y\leq 11$
4	(3,3) (5,-4)	$y=\frac{-7}{2}x+\frac{27}{2}$	D: $3\leq x\leq 5$
5	(5,-4) (11,-10)	$y=-1x+1$	D: $5\leq x\leq 11$
6	(11,-10) (11,-12)	$x=11$	R: $-12\leq y\leq 10$
7	(-11,-12) (11,-12)	$y=-12$	D: $-11\leq x\leq 11$
8	(0,-12) (-3,-14) (3,-14)	$y=\frac{-2}{3}\lvert x-0\rvert-12$	D: $-3\leq x\leq 3$
9	(3,-14) (3,-16)	$x=3$	R: $-16\leq y\leq -14$
10	(-3,-16) (3,-16)	$y=-16$	D: $-3\leq x\leq 3$
11	(-3,-14) (-3,-16)	$x=-3$	R: $-16\leq y\leq -14$
12	(-11,-10) (-11,-12)	$x=-11$	R: $-12\leq y\leq -10$
13	(-6,-8) (-5,-9) (-6,-10)	$x=-1(y+9)^2-5$	R: $-10\leq y\leq -8$
14	(-6,-8) (-7,-9) (-6,-10)	$x=1(y+9)^2-7$	R: $-10\leq y\leq -8$
15	(-5,-4) (-11,-10)	$y=1x+1$	D: $-11\leq x\leq -5$
16	(-3,3) (-5,-4)	$y=\frac{7}{2}x+\frac{27}{2}$	D: $-5\leq x\leq -3$
17	(-3,11) (-3,-12)	$x=-3$	R: $-12\leq y\leq 11$

Fig. 3: *Algebraic derivations for shuttle drawing*

calculations as I check for understanding. Once these drafts have been graded and returned, students begin work on final draft drawings (see Figs. 5 and 7). They also turn in a sheet algebraic derivations (including domain and range restrictions and accompanying work) for the graphical objects in their drawings (see Fig. 6).

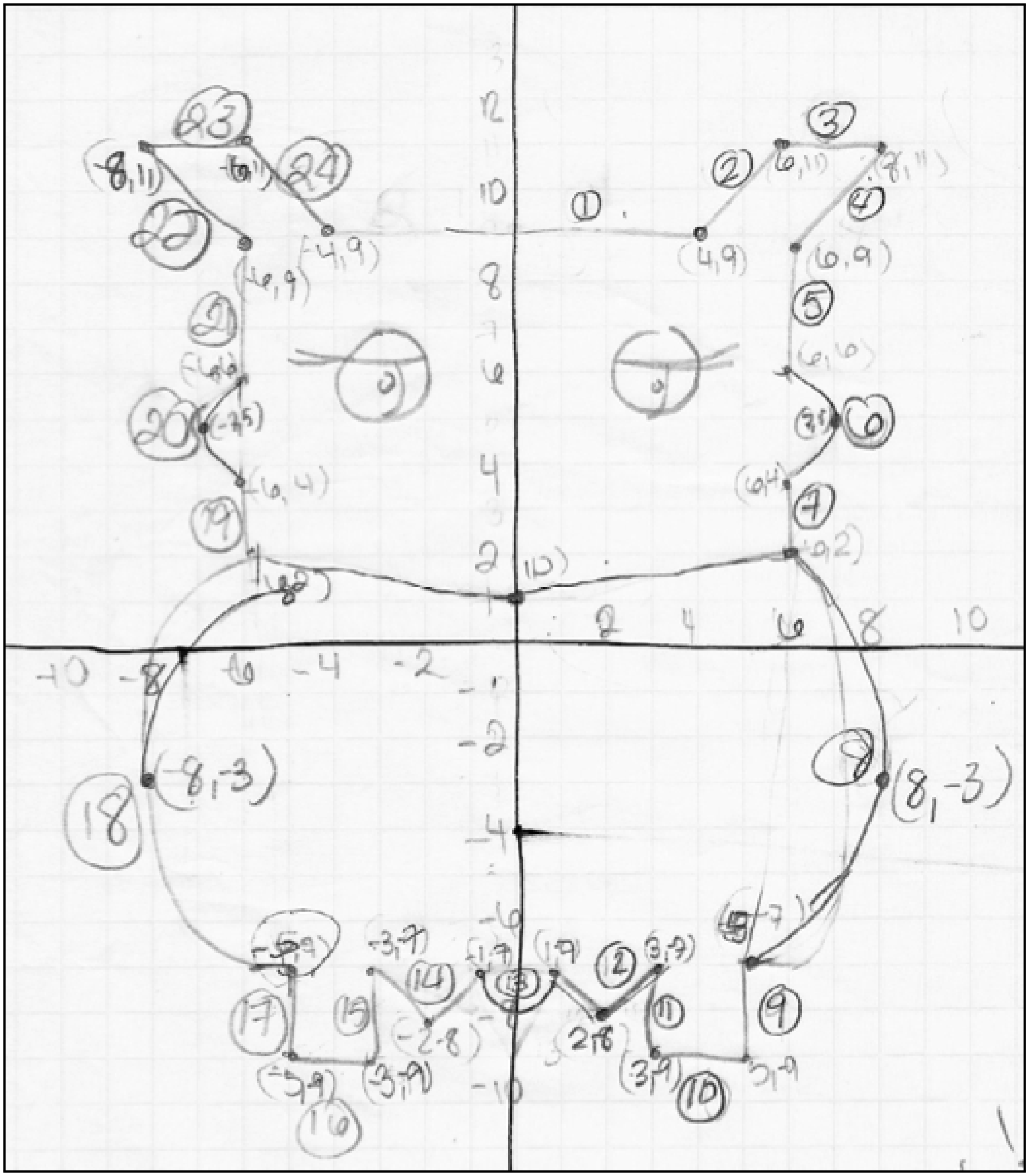


Fig. 4: *Student-created picture composed of numerous segments and curves*

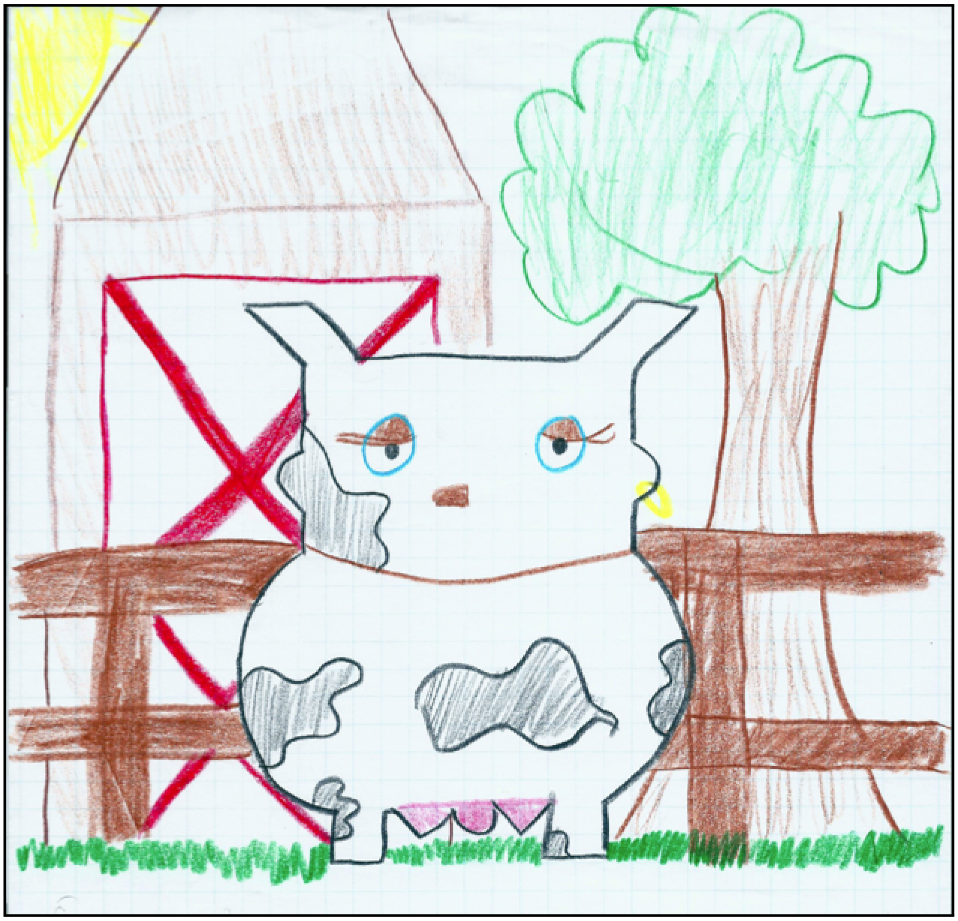


Fig. 5: *Student-created final draft drawing*

4 Some Suggestions for Grading

Through trial and error, I have developed some helpful hints to assist with the grading so it is not overwhelming. Develop a spreadsheet with all parts of the project that you intend to grade and grade each part as the project develops, rather than grading the entire project at the end. Checking for the accuracy of the mathematics is very important. Checking the rough draft to prevent any major problems later in the project is equally important. For the sketches, the students must have included at least one of each type of required function in their sketch; thus each function should be incorporated in the grading of the rough draft.

I suggest that you grade as many projects as you can at one sitting. When you do this, you notice common mistakes and can anticipate areas of potential trouble. If you grade only one or two at a time, you may spend more time trying to remember where the trouble areas are occurring. You must have the students turn in the rough draft with the final draft, to avoid having to regrade the entire project. With the rough draft in hand, one can focus on the incorrect areas to check that the students accurately corrected their mistakes.

Fig. 6: *Algebraic derivations (including domain and range restrictions) for numbered curves*

5 Tailoring the Project for Different Classes

Although the project originally described by Disher seems to be ideal for Algebra 2, I have found the project can be easily tailored to different courses from middle school Prealgebra to Trigonometry and even Precalculus. For algebra courses restrict the project to using only linear functions and vertical line segments. I introduce the project one day and in class have students work the next day. The entire project using only line segments takes only a few class days. This shortened version of the project can be assigned to Algebra 2 and Precalculus students as a review for lines within the first few weeks of the course. This allows students to review and is a foreshadowing to the more involved project later in the course. By changing the types of functions to conic sections and trigonometric functions, it is easy to use the same ideas presented creating pictures with those specific shapes.

Fig. 7: *Student-created final draft drawing*

6 Sample Assignment

Directions:

1. Use an entire sheet of graph paper and make an x- and y-axis. Label the axes.
2. Sketch your name using horizontal, vertical and diagonal line segments.
3. Number each segment. You need a minimum of 10 line segments.
4. Label all vertices with ordered pairs.
5. Write the function for each line segment. State the domain for horizontal and diagonal segments and the range for vertical segments.

What you are going to turn in:

- Rough draft of your name on graph paper with axes and vertices numbered
- Rough draft of functions showing all work to get a function for each segment. Use the same number on the segment to number the functions.
- Final draft of name without the axes or numbers for display. Color and any embellishments are encouraged.
- Final draft of functions and domains/ranges printed neatly or typed for display.

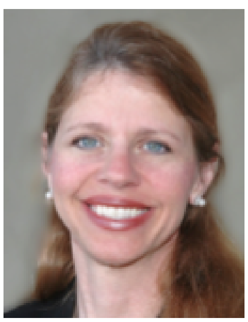
7 Conclusions

There are many extensions for this project. Students may incorporate functions (or relations) beyond those formally covered in class (e.g., the greatest integer function). Alternatively, students can program a computer or graphing calculator with functions and relations to reproduce pictures. First-year algebra students can work in pairs and give their partners symbolic representations of drawn graphs. By attempting to reproduce the drawing, the partnering student helps debug mistakes in their friend's work. Lastly, students can incorporate writing into the project, describing the motivation behind their artwork, the creative and mathematical processes involved in the project as well as their thoughts and feelings concerning the completion of the project. The beauty of the project is not only the student artwork to be displayed on the classroom walls, but also its versatility. The project can be easily tailored for use in any high school mathematics classroom.

References

Disher, F. (1995). Graphing art. *Mathematics Teacher, 88*, 124-128.

Marcus, R. & and Frey, J. T. (2003). Selecting quality tasks for problem-based teaching in *Teaching mathematics through problem solving: Grades 6-12* (Harold L. Schoen and Randall I. Charles (Eds.)). Reston, VA: National Council of Teachers of Mathematics, 55-67.



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